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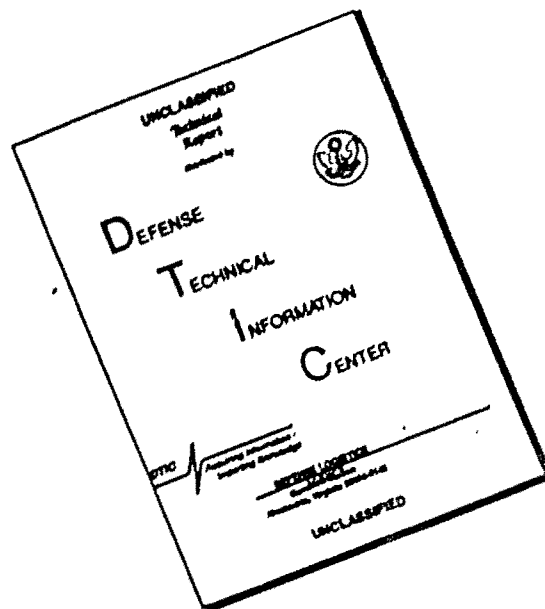
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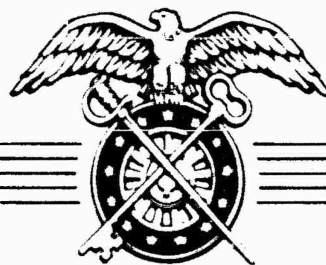
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TECHNICAL REPORT
EP-10

FC

A FACTORIAL ANALYSIS OF FOOT MEASUREMENTS



QUARTERMASTER RESEARCH & DEVELOPMENT CENTER
ENVIRONMENTAL PROTECTION DIVISION

JULY 1955

NATICK, MASSACHUSETTS

HEADQUARTERS
QUARTERMASTER RESEARCH & DEVELOPMENT COMMAND

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HEADQUARTERS QUARTERMASTER RESEARCH & DEVELOPMENT COMMAND
Quartermaster Research & Development Center, US Army
Natick, Massachusetts

ENVIRONMENTAL PROTECTION DIVISION

Technical Report
EP-10

A FACTORIAL ANALYSIS OF FOOT MEASUREMENTS

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FOREWORD

As long as our Army marches and fights on its feet, the design of adequate footwear for the soldier will remain a vexing problem for the Quartermaster Corps. We must provide footwear for all environments and situations in which the requirements may be to keep the soldier's feet warm or cool, dry or moist, lightly flexible or heavily "armored." Above all we must provide a reasonable fit, one which supports the none-too-rugged elements of the foot and ankle and yet a fit that does not interfere seriously with the inherent mechanical properties of the foot. All this must be accomplished on an extremity that turns a right angle and varies enormously in size and shape between men.

The solution of these problems and the multitude of others which plague the footwear designer and fabricator requires the services of many investigators in diverse occupations. This report represents the efforts of two men far removed from the profession of designing shoes to synthesize a large group of foot measurements into something condensed and yet meaningful to the footwear field. This report does not pretend to be the final word on the subject of foot dimensions or to exhaust the research available on the corpus of foot measurements from which it was derived, but it represents one additional step in our search for information and knowledge about that refractory appendage - the human foot.

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Acting Chief
Environmental Protection Division

Approved:

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QM Research & Development Command

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ABSTRACT

This report gives the results of a methodological study carried out for the purpose of investigating the nature of some of the critical factors determining the size of feet in Army personnel. Twenty-nine measures taken from the anthropometric data collected in the Fort Knox Foot Survey were factor analyzed.

Ten factors were extracted accounting for the intercorrelations between these 29 measures with relatively small residuals. The absolute size of the residual correlations after the extraction of the tenth factor was less than or equal to .06.

Eight of the ten rotated factors have been interpreted. This interpretation for each factor describes the nature of the underlying parameter responsible for the concomitant variation of the measures with large projections on the factor. Two factors were not interpreted.

The amount of the total variance for each measure accounted for by the ten factors varied from about 100 percent down to 11 percent. This indicates that at least one measurement (100 percent) shares all of its variance with one or more of the remaining 28 measures, and, consequently, could be relatively unimportant in the determination of shoe lasts. On the other hand, measures which share only 10 to 15 percent of their variance with other measures would have to be considered specifically in the proper fitting of shoes.

A FACTORIAL ANALYSIS OF FOOT MEASUREMENTS

1. Introduction

a. Purpose of the study.

The anthropometric data collected in the Fort Knox Foot Survey¹ were factor analyzed for the purpose of obtaining a clearer understanding of the basic factors determining the size of feet in Army personnel.

b. The factorial method.

1) The purpose of factor analysis: Thurstone² has stated that the factorial methods were developed primarily for the purpose of identifying the principal dimensions or categories of mentality; but the methods are general, so that they have been found useful for other psychological problems and in other sciences.

The factorial methods are sometimes employed by an investigator to test an hypothesis when he is unable to devise some other critical test. They may also be used to explore a range of phenomena to determine the underlying constructs. For example: From the available data we have selected 29 different physical measurements. These measurements were made on a large number of soldiers.

The measures are intercorrelated over this sample of soldiers; the purpose was to find that a relatively small number of factors will suffice to account for the correlations obtained between the observed measures. Our thesis from the beginning of the study was that the number of basic parameters involved would be considerably fewer than the number of measures.

2) Factor analysis and multiple regression: In the sense that one is attempting to predict the dependent variable from two or more independent variables, there is no direct relation between factor analysis and a multiple regression technique. In the factorial problem an attempt is made to determine the factors from the test

¹ Freedman A., E. C. Huntington, G. C. Davis, R. B. Magee, V. M. Milstead, C. M. Kirkpatrick. Foot Dimensions of Soldiers, Proj. No. T-13 SGO No. 611 Survey of Foot Measurements of Proper Fit of Army Shoes (Third Partial Report). Armored Medical Research Laboratory, Ft. Knox, Ky., 11 Mar 1946.

² Thurstone, L. L. Multiple-Factor Analysis. A Development and Expansion of the Vectors of Mind. Univ. of Chicago Press, Chicago, Ill., 1947.

scores or other measures. The factors could be considered to be the dependent variables and the test scores the independent variables. Generally there is no distinction made between dependent and independent variables in factor analysis. A close parallel exists when one knows the factor scores of an individual and attempts to predict his performance on a test of known factorial composition. The factor scores would represent the independent variables while the predicted score would be the dependent variable.

3) The fundamental equations of factor analysis: The basic equation of factor analysis is written

$$s_{ji} = c_{j1}x_{1i} + c_{j2}x_{2i} + c_{j3}x_{3i} + \dots + c_{jq}x_{qi}$$

Where s_{ji} is the standard score of individual i in measure j ; x is the standard score of individual i in each of the uncorrelated reference parameters; and the c is the weight assigned to the standard score in the reference parameters for the determination of the observed standard score s_{ji} . The equation is written to represent q terms in the right-hand member and this number of reference parameters should be relatively small compared with the number of measures, n , in the whole set of measurements.

The c is called the measure coefficient or factor loading. To determine this we start with a score matrix S of order $n \times N$, where the elements of S are s_{ji} , the standard score of individual i on measure j . N refers to the number of individuals and n denotes the number of measures. The complete correlation matrix is defined as the symmetric matrix of intercorrelations between the measures with unity in the diagonals. This matrix is denoted R_1 and is of order $n \times n$. In matrix notation

$$R_1 = 1/N SS'$$

with elements $r_{jk} = 1/N \sum_i s_{ji}s_{ki}$

The complete correlation matrix R_1 has unity in the diagonal entries. For the purpose of this study this is not desired; the self correlation of any measure is unity. This implies an attempt to account for the entire variance of the measure. For a study such as this we are interested only in that part of the variance of each measure that is shared with other tests. We are not interested in the part of the variance which is specific to the measure or which is due to error. That part of the variance of a measure which is shared is called the "communality" and is designated h^2 . That part

which is specific to the measure plus that part which is due to error is termed "uniqueness." The communality plus the uniqueness is equal to unity.

When the communalities are placed in the diagonals of a correlation matrix, the matrix is known as the "reduced correlation matrix" and is designated as R. This is the matrix which has been factored in this study. The factor matrix derived by the centroid method of factoring is denoted F_0 . The factor matrix post-multiplied by its transpose will reproduce the reduced correlation matrix with only slight discrepancies. In matrix notation

$$F_0 F_0' = R$$

2. The Data

The anthropometric data collected in the Fort Knox Foot Survey were used in this analysis. In this survey 41 measurements were taken for each of 5,571 White and 1,200 Negro soldiers. Our analysis has been restricted to the population of White soldiers, since it has been previously noted Negro feet are somewhat aberrant in certain dimensions.

a. Consideration of age as a variable.

Age was one of the 41 measures for which data were gathered. The distribution of "age" for the 5,571 White soldiers is given in Table I. Since this distribution is far from "normal" and not representative of the soldier population, it was decided to select one age group for our analysis. The 18-year age group, which represented almost half of the total series, was selected for our analysis. Age is therefore eliminated as a variable in this analysis.

TABLE I: AGE DISTRIBUTION FREQUENCY FOR 5,571 WHITE SOLDIERS
(Fort Knox Foot Survey)

<u>Age</u> <u>(Years)</u>	<u>Fre-</u> <u>quency</u>	<u>Age</u> <u>(Years)</u>	<u>Fre-</u> <u>quency</u>	<u>Age</u> <u>(Years)</u>	<u>Fre-</u> <u>quency</u>	<u>Age</u> <u>(Years)</u>	<u>Fre-</u> <u>quency</u>
17	2	23	72	29	570	35	23
18*	2,741	24	62	30	298	36	13
19	385	25	91	31	48	37	22
20	168	26	54	32	49	38	2
21	129	27	186	33	42		
22	81	28	505	34	28		

*This age group was used in present report.

b Selection of 29 variables to be analyzed.

The 40 remaining variables were inspected to insure that they were independent and continuous measures. Eleven of these 40 measures were considered to be unsuitable; these 11 are listed below with a brief explanation of why they were rejected. The page reference given, in parentheses, refer to the report "Foot Dimensions of Soldiers."

1) Place enlisted: This measure is merely a code number assigned to the individual depending upon his geographic location at the time of induction (page 27).

2) Clinical evaluation: The categories used do not constitute a quantitatively continuous scale (page 154).

3) Identification of most elevated toe: This measure is not a continuous variable. Values from one to five were recorded, depending upon which toe had the greatest elevation (page 6).

4) Toe length: Foot length is equal to the sum of ball length and toe length. Therefore given the ball length and foot length, toe length is determined. This measure is experimentally dependent on the other two (page 58).

5) Contour and orientation of toes (by template): This measure is not a continuous variable (page 74).

6) Foot length (stick): This measure was used to check the photographic measurements of foot length. It was done for approximately two-thirds of the population (page 21).

*7) Difference: right-left for measure of dorsal arch height
(page 113).

8) Difference: right-left for measure of ball girth (page 89).

9) Difference: right-left for measure of foot length (page 41).

10) Difference: right-left for measure of ball length (page 43).

11) Difference: right-left for measure of foot breadth-diagonal
(page 80).

* Measures 7 through 11 were all rejected for the same reason, i.e., they are concerned with differences between the right and left foot. For each measure the amount of the difference between right and left foot was represented graphically. As a check, first and second measurements were made on the same foot for each measure. In every instance for these five measures the distribution of differences between first and second measurements and the differences between right and left foot were similar enough to suggest that errors in measuring technique provided a large portion of the observed right and left foot differences

Accordingly, a total of 29 variables were used in the present analysis, and they are listed in Table II and represented graphically in Figure 1.

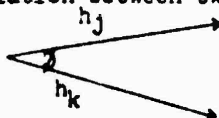
TABLE II: VARIABLES USED IN FOOT ANALYSIS

Identify- ing No.	Measure	Identify- ing No.	Measure
1	Height (in Centimeters)	18	Breadth of Three Forward Toes
2	Weight (in Kilograms)	19	Foot Breadth-Diagonal
3	Height of Great Toe Tip	20	Angle Line I to J
4	Toe Height	21	Ball Breadth on Horizontal
5	Ball Height	22	Width (Center Line to Medial Border)
6	Plantar Arch Height	23	Flare (Ratio)
7	Dorsal Arch Height	24	Breadth of Instep
8	Outside Ball Height	25	Proportion of Sole in Contact with Ground
9	Ankle Length	26	Lateral Foot Contour by Template
10	Posterior Heel Contour	27	Heel Breadth
11	Ball Girth	28	Instep Girth
12	Ankle Girth	29	Outside Ball Length (Diagonal)
13	Lower Leg Girth		
14	Foot Length		
15	Ball Length		
16	Fifth Toe Length		
17	Outside Ball Length		

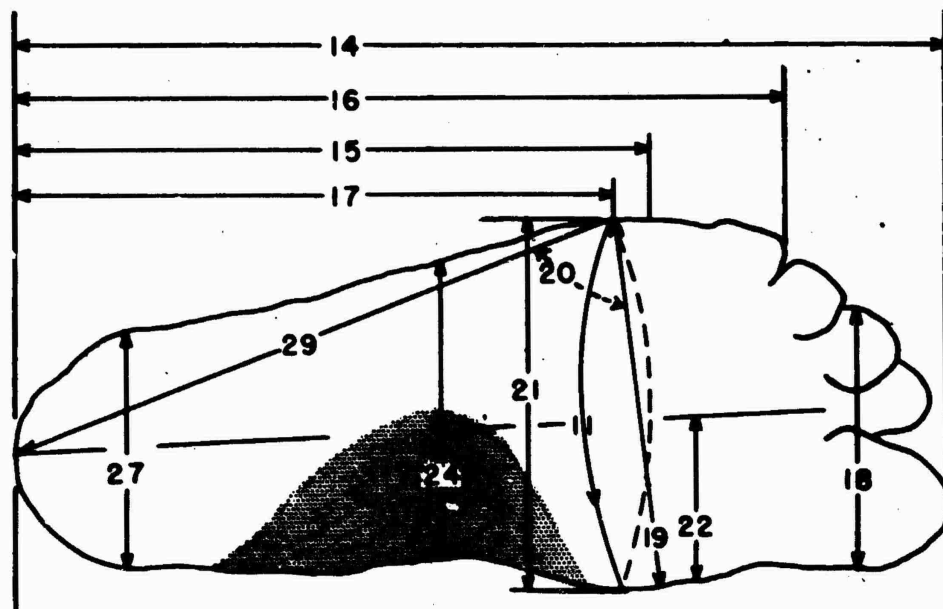
For these 29 measures, the sample of 18-year-old soldiers was checked to be sure that complete records on all 29 measures were available. One hundred and one individual records were rejected because they were incomplete, thus, the sample of 18-year-old soldiers used in this study consisted of 2,640 cases.

c. Intercorrelations and factoring.

Product-moment correlation coefficients were computed for all pairs of the 29 tests. Appendix A gives these 406 intercorrelations below the diagonal. Only one-half of the correlation matrix is given since the upper half is symmetric to the lower half. The correlation coefficient can be represented geometrically. In the diagram below the correlation between two measures j and k is shown. The correlation between the two measures is equal to $h_j h_k \cos \theta$ where the h represents the lengths of the vectors and θ shows the angle of separation.



If two measures have zero correlation the angle between them must be 90 degrees. In a three-dimensional problem we could construct the correlation matrix using a series of wires to represent the vectors.



$$23 = \frac{22}{21}$$

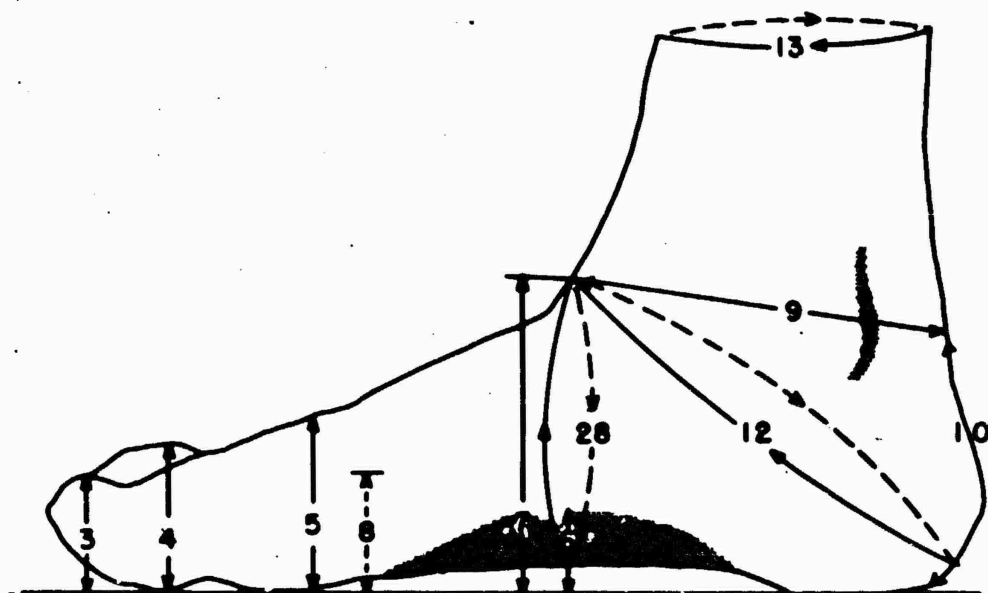


Figure 1. Foot measure used (as listed in Table II).

In this study we have more than three underlying parameters involved. To analyze these data given in the table of correlations we must insert a reference frame on the vector configuration. This is done by factoring. One axis at a time is located and the projections of the test vectors on it determined. Each successive axis is mutually orthogonal to all others. We continue inserting axes until the residual correlations are considered to be negligible.

For locating the axes in this study the complete centroid method was used. Ten centroid factors were extracted. The absolute value of the largest residual correlation remaining after the extraction of ten factors was .06. One half of the tenth factor residual correlation matrix appears above the diagonal in Appendix A. As for the correlation matrix, only half of the table is shown since the other half is symmetric. The centroid factor matrix F appears in Appendix B. A frequency distribution of the tenth factor residual correlations is given in Appendix C.

d. The rotation of axes.

These reference axes were rotated to simple structure using radial rotation methods. In rotating we attempt to maximize the number of near zero projections on each reference axis. The rotated reference axes are no longer necessarily orthogonal. The transformation matrix Λ which transforms the centroid factor matrix F into the rotated oblique factor matrix V is given in Appendix D. The cosines of the angles between the oblique reference axes are given in Appendix E. These values are obtained from the matrix multiplication. The final oblique factor matrix appears in Appendix F.

A set of the final plots between pairs of oblique reference axes is included as Appendix G. There are 45 of these diagrams. The plots are made using orthogonal axes. It has been found through experience that although some distortion of the picture is present when these diagrams are plotted as if the axes were orthogonal, this distortion is not serious when the cosines are small.

The oblique factor matrix is the principal objective of the factorial computations. This is the matrix on which the interpretation of the factors is based. In the interpretation of each factor we usually consider only those tests which have sizable projections. For each factor we can select those variables which have a projection equal to or greater than $\pm .20$ and ignore the remaining variables. The pattern of these measures with high loadings is called the factor pattern. The factor pattern for this study is given in Appendix A. One does not consider the variables with low projections in interpreting the factors. However, the fact that a number of measures have near-zero projections on a given factor must be consistent with the final interpretation given. For example: Consider a factor which has been interpreted as foot length. If a measure such as ball length had a near-zero projection on this factor, we would consider the interpretation questionable. In this way we check to see that our interpretations for each factor are consistent.

3. Results and Discussion (Interpretation of the Common Factors)

The identification of the underlying dimensional parameters which best describe the factors extracted here has been accomplished by examination of those factor loadings with values of .20 or greater. This is an arbitrary limit, but the descriptive clues to the factor parameters are presumably in the high factor-loaded measures rather than those near the arbitrary lower limit. These values have been isolated for convenience in Table III and collected by factor along with a description of each measure below. It is possible to differ with the verbal identifications we have given each of the factors and it may be that scientists with greater experience with foot problems will wish to re-define our definitions.

a. Factor A.

The foot measures in the Factor Pattern for Factor A are listed below in order of importance (large to small).

<u>No.</u>	<u>Measure</u>	<u>Proje- ction</u>
17	Outside ball length	.77
29	Outside ball length (diagonal)	.76
16	Fifth toe length	.70
14	Foot length	.68
15	Ball length	.63
1	Height (of subject)	.50
9	Ankle length	.34
12	Ankle girth	.25

These 8 measures have values greater than .20. The remaining 21 variables as shown in Appendix F have projections less than $\pm .07$. Since the individual's height (1) appears on this factor, the underlying parameter seems to be bone length. It is interesting to note that while tall men will tend to have longer feet, there appears to be no relation between stature and foot width. The measures of ankle length (9) and ankle girth (12) seem less obviously related to bone length than the other measures. It seems possible that if the heel bone is somewhat longer also, then these two measures would vary with bone length.

b. Factor B.

<u>No.</u>	<u>Measure</u>	<u>Proje- ction</u>
23	Flare (ratio) = 22/21	.90
22	Width center line to medial border	.89
21	Ball breadth on horizontal	.26
16	Fifth toe length	.23
17	Outside ball length	.23

TABLE III

FACTOR PATTERN*

Foot Measure	F A C T O R S									
W	A	B	C	D	E	F	G	H	J	K
1	50									21
2							20	43		36
3						23				
4									30	27
5				36		24			22	
6				70						
7				68						
8						35			31	
9	34						38			20
10									22	
11			45							
12	25						33		24	
13							42	32		
14	68									
15	63									
16	70	23			25					
17	77	23			41					
18						35				
19			48							
20			-20		-70					
21		26	50		26					
22		89								
23		90								
24				-41						
25				-40				23		
26			37		25	-33	21			
27								20	27	
28								32		
29	76				46					

*Decimals have been omitted. Listed here are variables which have a projection equal to or greater than ± 20 , as shown in Appendix F.

These 5 measures have projections greater than .20. The remaining 24 variables have projections less than $\pm .10$ (Appendix F). This factor is essentially a doublet. Measures 22 and 23 are experimentally dependent. These 2 variables have an intercorrelation of .97.

The factor has been interpreted as foot flare. The first 3 measures clearly relate to the flaring of the foot in the region of the first and fifth metatarso-phalangeal joints. Measures 16 and 17 related to the outside length of the foot. If this length is longer than average, it may indicate that the line joining the first and fifth metatarso-phalangeal joints is more nearly perpendicular to the long axis of the foot. This would result in a greater horizontal ball breadth and consequently greater foot flare.

c. Factor C.

<u>No.</u>	<u>Measure</u>	<u>Projection</u>
21	Ball breadth on horizontal	.50
19	Foot breadth (diagonal)	.48
11	Ball girth	.45
26	Lateral foot contour by template	.37
20	Angle line I to J	-.20

The common element seems to be ball breadth. The lateral foot contour could be expected to have a greater angle of curvature with the broader foot. Variable 20 is the angle measured at the fifth metatarso-phalangeal joint between the center of the heel and the first metatarso-phalangeal joint. With increase in ball breadth this angle could be expected to decrease in size; this accounts for its negative projection on this factor.

d. Factor D.

<u>No.</u>	<u>Measure</u>	<u>Projection</u>
6	Plantar arch height	.70
7	Dorsal arch height	.68
24	Breadth of instep	-.41
25	Proportion of sole in contact with ground	-.40
5	Ball height	.36

The underlying parameter seems to be arch height. With a well arched foot the measures 5, 6, and 7 will be relatively large. The breadth of the instep will be small and the proportion of the sole in contact with the ground will be small. When the arch is broken down, as in the instance of flat feet, the converse relationships will hold.

Variables 5, 6, and 7 have an inverse relationship to variables 24 and 25; this accounts for the fact that variables 24 and 25 have negative projections on Factor D.

e. Factor E.

<u>No.</u>	<u>Measure</u>	<u>Projection</u>
20	Angle line I to J	-.70
29	Outside ball length (diagonal)	.46
17	Outside ball length	.41
21	Ball breadth on horizontal	.26
26	Lateral foot contour by template	.25
16	Fifth toe length	.25

The common factor involved seems to be the angular orientation of the metatarsal heads. Variable 20 has the most important loading on this factor. The fact that its loading is a -.70 instead of a +.70 is due to the arbitrary definition of the positive direction of the reference vector. It could just as well have been given as +.70 while the other five variables would be given as having negative projections. The angle line I to J is inversely related to the other five measures. The wider a foot becomes relative to its length, the smaller the angle becomes. The longer the outside ball length both diagonal and vertical, the smaller the angle will become. Since the lateral foot contour varies with outside ball length, an increase in the angle of contour will be accompanied by a decrease in the angle line I to J. In converse fashion, the angular orientation of the metatarsal heads would tend to become larger as these other variables decrease in relative size.

f. Factor F.

<u>No.</u>	<u>Measure</u>	<u>Projection</u>
18	Breadth of three forward toes	.35
8	Outside ball height	.35
26	Lateral foot contour by template	-.31
5	Ball height	.24
3	Height of great toe tip	.23

The underlying element seems to be toe size. No measures of toe length alone were included in the variables so it cannot be said that length of toe is also basic. However, since breadth of the three forward toes (18) and height at the tip and joint of the first toe and height at the joint of the fifth are included it seems possible that toe length is also indicated. Lateral foot contour by template (26) will vary inversely with toe length on the lateral side of the foot. Long fourth or fifth toes will produce a small angle of curvature; this would account for its negative projection on this factor.

g. Factor G.

<u>No.</u>	<u>Measure</u>	<u>Proje- ction</u>
13	Lower leg girth	.42
9	Ankle length	.38
12	Ankle girth	.33
26	Lateral foot contour by template	.21
2	Weight	.20

The first three measures seem to indicate that this factor is related to lower leg size. Weight should be somewhat related to lower leg size; however, it seems unlikely that lateral foot contour would be related to lower leg size. The interpretation of this factor is not wholly consistent.

h. Factor H.

<u>No.</u>	<u>Measure</u>	<u>Proje- ction</u>
2	Weight	.43
28	Instep girth	.32
13	Lower leg girth	.32
25	Proportion of sole in contact with ground	.23
27	Heel breadth	.20

This seems to be a weight factor. Fleshy individuals should tend to have higher values in the other four measures.

i. Factors J and K.

The remaining two factors (J and K) have been left without interpretation. There seems to be no underlying common thread which would seem to link the measures. These measures which have projections greater than + .20 on these two factors J and K are given in Table III.

The column labelled h^2 (communality) in Appendix I can be considered as giving an indication of the total variance of each measure that has been accounted for by the common factors. All of the variance of measure 22 has been accounted for. Since the flare ratio is in common use, variable 22 could be omitted from future analyses. The remaining measures range from .96 to .11. In those instances where the communality is very low, the variance is largely specific to that measure and little is shared with the other measures analyzed.

4. Summary

Twenty-nine measures selected from those included in the Fort Knox

Foot Survey were factor analyzed. Ten factors were needed to account for the inter-correlations between these selected variables. Eight of the rotated factors have been given interpretation. These interpretations are:

a. Factor A seems to be related to bone length. The height of the subject as well as measures of foot length appear on this factor.

b. Factor B is related to the foot flare. This factor is essentially identified by two measures. The remaining three measures with projections greater than .20 on this factor are markedly smaller than the first two.

c. Factor C is interpreted as ball breadth. Measure 20 has a negative projection on this factor. This measure is the angle line I to J. It seems reasonable that this measure would vary inversely with ball breadth.

d. Factor D seems to involve arch height. On this factor the dorsal and plantar arch measures have sizable loadings. In contrast the measures for breadth of instep and proportion of the sole in contact with the ground have large negative projections.

e. The angular orientation of the metatarsal heads seems to be the determining parameter in Factor E. Again the factor has measures with both positive and negative projections.

f. Factor F seems to be a factor of toe size. The breadth of the three forward toes and measures of ball and toe height have the large projections.

g. The interpretation of Factor G is not wholly consistent. While four of the measures seem to be related to lower leg size, the measure for lateral foot contour does not appear to be related.

h. Factor H appears to be a weight factor. The measures having projections on this factor are directly related to the weight of the individual.

i. No tentative interpretations have been made for the remaining two rotated factors (J and K).

The percentage of variance for each of the 29 measures analyzed varies from 100 down to 11 percent. Seven measures did not account for half of their variance. Measures with such small communalities are poorly described in terms of the analysis carried out in this study.

These measures if considered important in determining shoe lasts require additional study. It may well be that these measures are highly correlated with other foot dimensions which were not analyzed in this

study. From the evidence obtained in this report, it must be concluded that the measures with relatively low communalities must be considered individually in the determination of the foot size of Army personnel.

5. Acknowledgements

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We also acknowledge the help of the following members of the Psychometric Laboratory who did much of the computing and preparation of this report: John D. Kelton, David N. Crissman, Edward W. Kearsley, Maurice Hall, Lorenza Clinard, Colin J. Williams, Robb Taylor, and Elizabeth P. Cupp.

Appendix A
INTERCORRELATIONS FOR PAIRS OF 29 TESTS*
(Below Diagonal Intercorrelations between Twenty-nine Foot Measures)
(Above Diagonal Tenth Factor Residual Correlations)

Foot Measure	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
1																													
2	02																												
3	00	00																											
4	00	00	00																										
5	00	00	00	00																									
6	00	00	00	00	00																								
7	00	00	00	00	00	00																							
8	00	00	00	00	00	00	00																						
9	00	00	00	00	00	00	00	00																					
10	00	00	00	00	00	00	00	00	00																				
11	00	00	00	00	00	00	00	00	00	00																			
12	00	00	00	00	00	00	00	00	00	00	00																		
13	00	00	00	00	00	00	00	00	00	00	00	00																	
14	00	00	00	00	00	00	00	00	00	00	00	00	00																
15	00	00	00	00	00	00	00	00	00	00	00	00	00	00															
16	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00														
17	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00													
18	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00												
19	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00											
20	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00										
21	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00									
22	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00								
23	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00							
24	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00						
25	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00					
26	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00				
27	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00			
28	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00		
29	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	

*Foot Measures based upon 1944

APPENDIX B

Centroid factor Matrix F*

A X E S

Foot Meas- ures	<u>I</u>	<u>II</u>	<u>III</u>	<u>IV</u>	<u>V</u>	<u>VI</u>	<u>VII</u>	<u>VIII</u>	<u>IX</u>	<u>X</u>	Communa- lity h^2
1	62	-12	45	07	12	-14	-07	03	-04	-13	66
2	76	17	04	-12	-12	06	-23	17	04	-17	75
3	24	18	-05	-02	-02	06	19	05	-10	06	15
4	47	11	-07	-10	04	15	06	-11	09	-18	33
5	50	06	09	-41	07	16	16	05	-09	09	50
6	13	-38	17	-56	21	12	13	-07	11	13	61
7	36	-33	17	-58	17	-10	18	27	19	12	80
8	47	19	-16	-05	-11	18	24	-06	-18	-10	43
9	80	16	31	14	04	14	-18	-19	-19	06	91
10	09	14	-09	14	-03	03	11	-14	13	12	12
11	86	21	-20	-18	16	-20	-13	-08	-04	-05	95
12	87	19	22	-03	-06	13	-07	-10	03	11	89
13	78	20	10	-19	-08	12	-27	09	-04	06	80
14	78	07	43	33	12	-18	09	01	-04	06	97
15	68	23	46	37	11	-22	04	-07	12	02	95
16	76	-28	29	33	-08	-07	10	06	01	05	88
17	70	-37	27	41	-23	-05	13	01	-04	02	94
18	47	20	-11	-12	14	-07	12	11	-21	-05	38
19	79	17	-26	-05	22	-28	-13	-10	12	02	89
20	-15	42	34	22	49	09	-07	06	20	-04	66
21	83	-13	-39	-06	05	-23	-08	-08	-06	11	95
22	37	-62	-48	23	27	29	-12	13	-06	06	100
23	19	-65	-41	25	27	35	-08	19	-06	06	93
24	58	61	-19	19	-11	-14	-04	05	-06	09	83
25	15	22	-27	31	-23	-04	-10	07	11	-06	32
26	06	-19	-05	-16	-07	-17	-27	-35	04	12	31
27	65	27	-14	-03	-08	07	11	07	18	06	58
28	82	25	-08	-14	-10	-07	-04	26	09	07	86
29	74	-20	31	33	-31	17	12	-07	-08	-03	94

*Decimals have been omitted.

A P P E N D I X C

Distribution of Tenth Factor Residuals* from Appendix A

<u>Residual</u>	<u>Frequency</u>	<u>Residual</u>	<u>Frequency</u>
-06	4	00	196
-05	12	01	134
-04	18	02	80
-03	38	03	38
-02	100	04	10
-01	172	05	10
		06	0

A P P E N D I X D

Transformation Matrix

F A C T O R S

<u>Axes</u>	<u>A</u>	<u>B</u>	<u>C</u>	<u>D</u>	<u>E</u>	<u>F</u>	<u>G</u>	<u>H</u>	<u>J</u>	<u>K</u>
I	282	150	140	090	131	059	098	119	149	152
II	-333	-450	-069	-329	-429	190	080	050	126	023
III	562	-372	-353	274	-106	-064	220	000	-097	106
IV	524	415	-132	-600	-060	-065	-181	-119	-073	-120
V	-117	387	257	283	-736	191	-226	-438	-060	-099
VI	-257	374	-642	135	-295	106	411	247	601	286
VII	356	-129	-213	408	161	583	-613	-476	467	-248
VIII	-111	264	-481	-143	-280	192	-111	479	-502	-108
IX	-030	-082	-280	405	-224	-686	-186	455	335	-029
X	000	286	000	000	000	-239	509	-231	000	-886

A P P E N D I X E

Reference Vector Cosines C=

<u>Fac-tors</u>	<u>A</u>	<u>B</u>	<u>C</u>	<u>D</u>	<u>E</u>	<u>F</u>	<u>G</u>	<u>H</u>	<u>J</u>	<u>K</u>
A	1.001									
B	-014	1.000								
C	-084	-089	1.000							
D	056	-153	-128	1.001						
E	346	-244	257	-073	999					
F	041	-003	-017	-035	-041	1.000				
G	-250	099	-131	-167	-015	-354	999			
H	-294	-048	-499	-103	-062	-476	274	1.000		
J	-028	-047	-297	451	-012	042	-016	-099	1.000	
K	-105	-56	-098	020	-009	092	-080	405	130	1.000

*Decimals have been omitted.

A P P E N D I X F

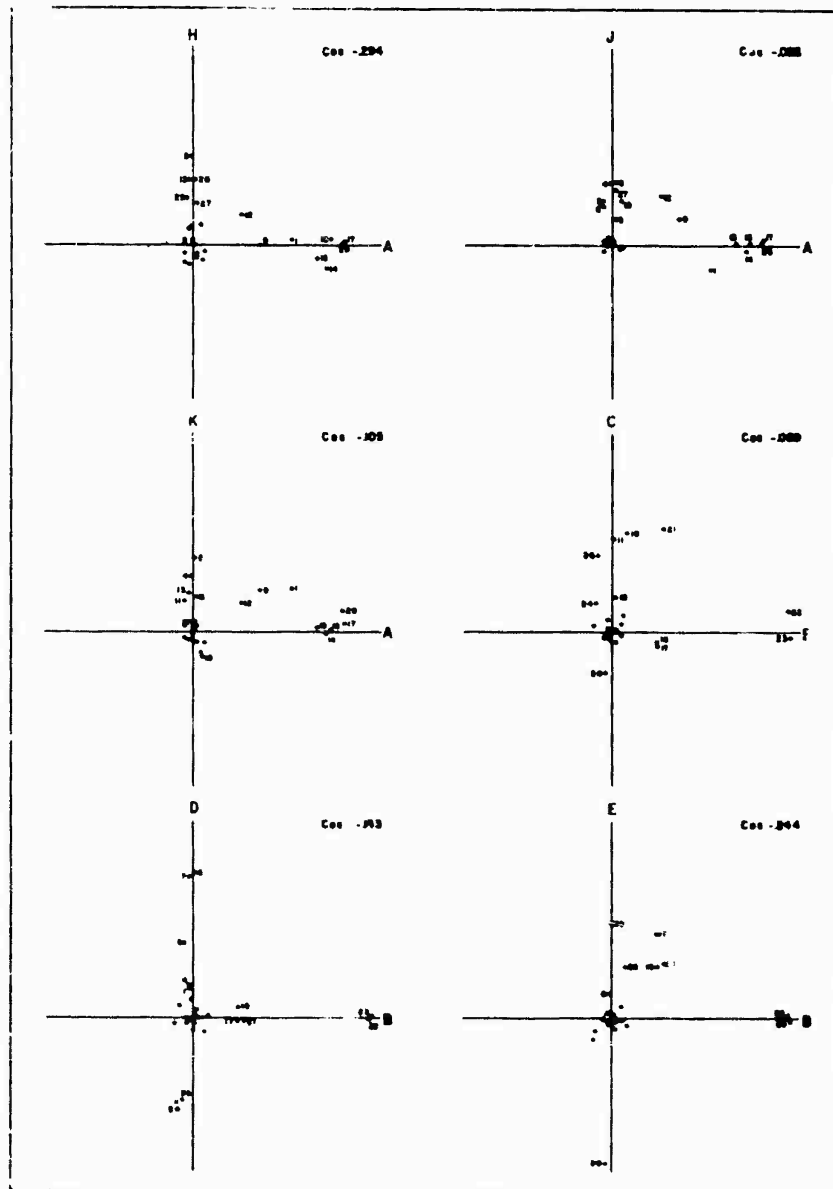
Oblique Factor Matrix V*

F A C T O R S

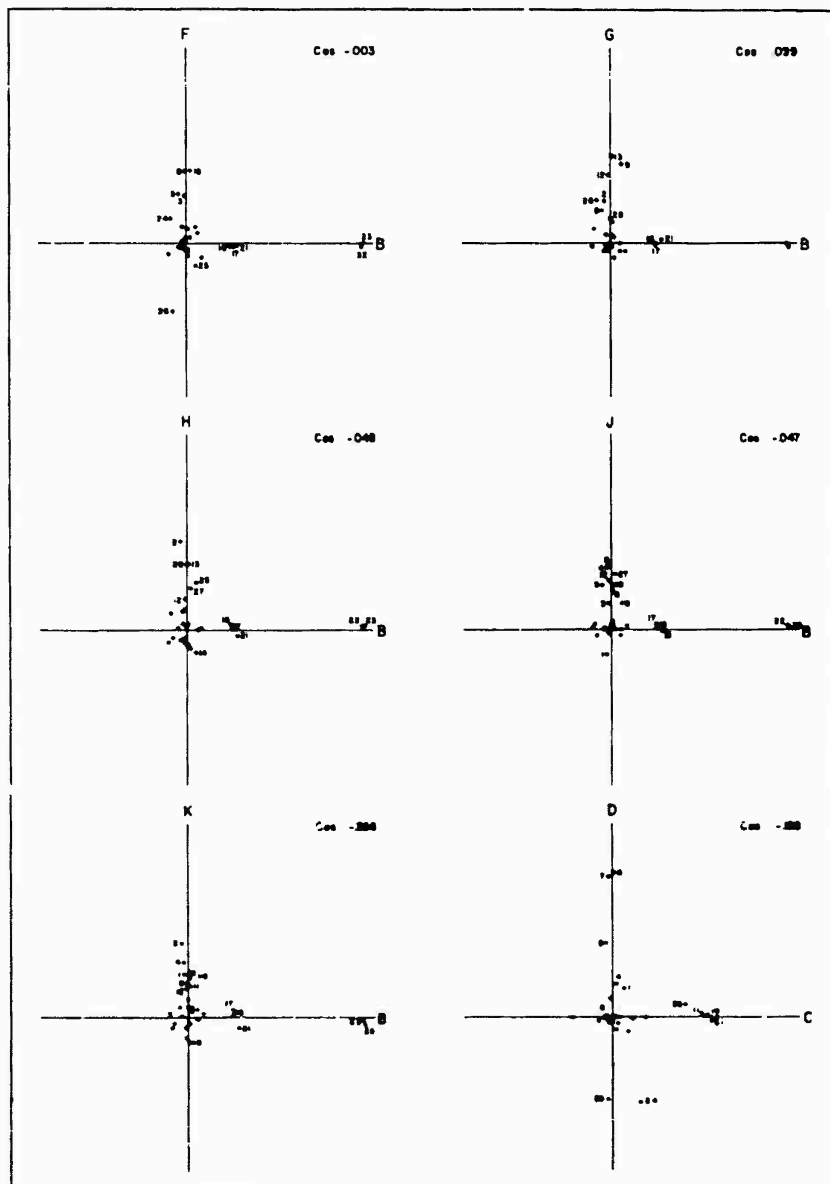
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1	50	-02	06	14	03	01	04	03	-12	21	66
2	01	-03	-02	-03	01	-02	20	43	00	36	75
3	02	-01	-04	-01	00	23	-01	-06	13	-05	15
4	-01	-02	02	16	-03	08	-02	09	30	27	33
5	-04	-04	-03	36	-01	24	16	01	22	05	50
6	-04	01	01	70	02	-03	10	-08	19	-03	61
7	04	-01	-02	68	02	00	-04	10	-02	-10	80
8	02	-01	00	-01	12	35	-02	-04	31	17	43
9	34	06	08	-07	-01	05	38	00	13	20	91
10	05	01	00	-03	-01	-06	-02	-07	22	-12	12
11	-04	01	45	02	01	07	04	03	02	15	95
12	25	-01	-01	09	03	-03	33	15	24	14	89
13	-02	01	01	00	-02	-04	42	32	04	19	80
14	68	05	04	00	-01	08	00	-11	-03	-01	97
15	63	-09	03	-03	-10	-05	-02	-06	01	02	95
16	70	23	-05	05	25	-02	00	03	01	01	88
17	77	23	-07	-02	41	-02	-01	02	03	04	94
18	-02	02	17	00	-05	35	-07	-09	00	05	38
19	00	08	48	01	-04	-07	-04	01	02	02	89
20	01	-03	-20	00	-71	-02	-04	-05	01	01	66
21	06	26	50	-02	26	-01	02	-03	-01	-05	95
22	00	89	10	-01	02	-02	00	02	02	-02	100
23	00	90	-03	01	-02	00	-02	02	01	-04	93
24	-02	-08	14	-41	-06	12	07	08	03	-04	83
25	-03	07	-02	-40	06	-11	-04	23	00	04	32
26	-04	-07	37	06	25	-33	21	-04	-03	-03	31
27	02	02	-05	04	-02	03	03	20	27	03	58
28	01	00	02	00	01	03	12	32	02	05	86
29	76	00	02	-06	46	01	00	01	01	09	94

APPENDIX G

Plots between Pairs of Oblique Reference Axes

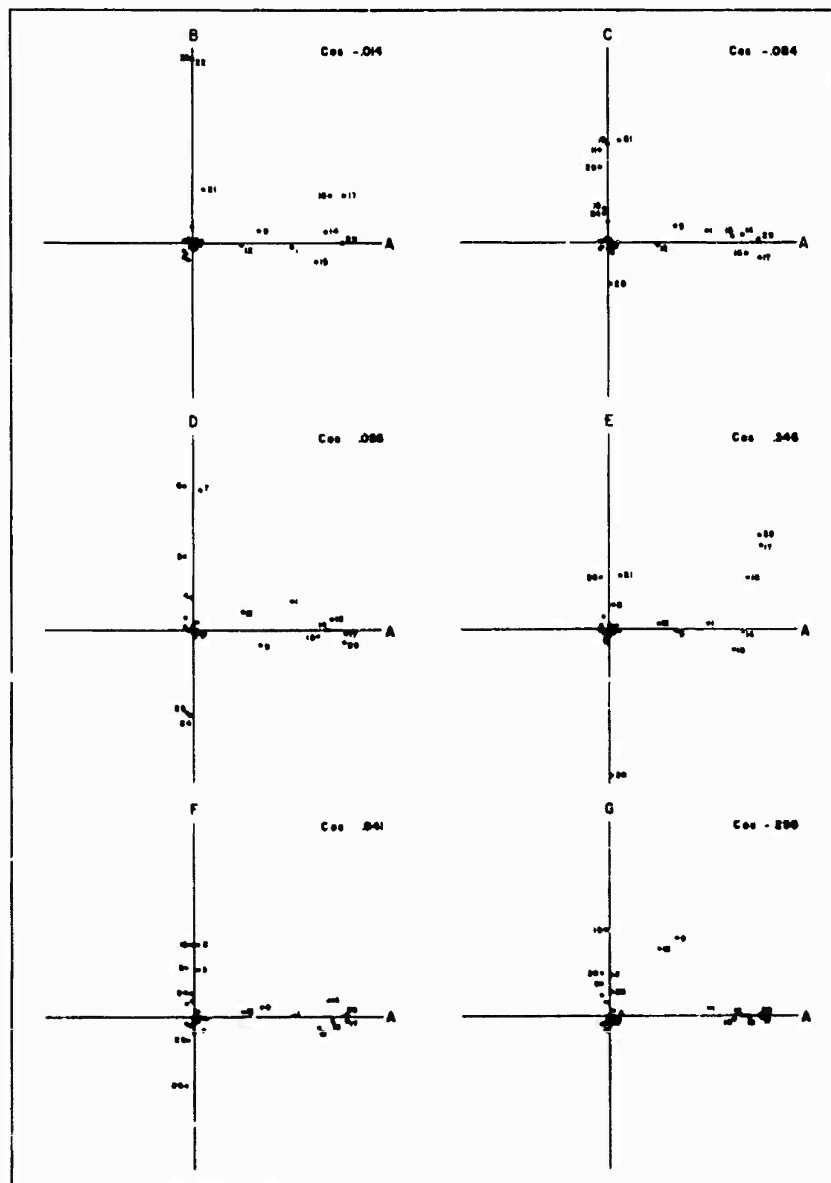


APPENDIX G
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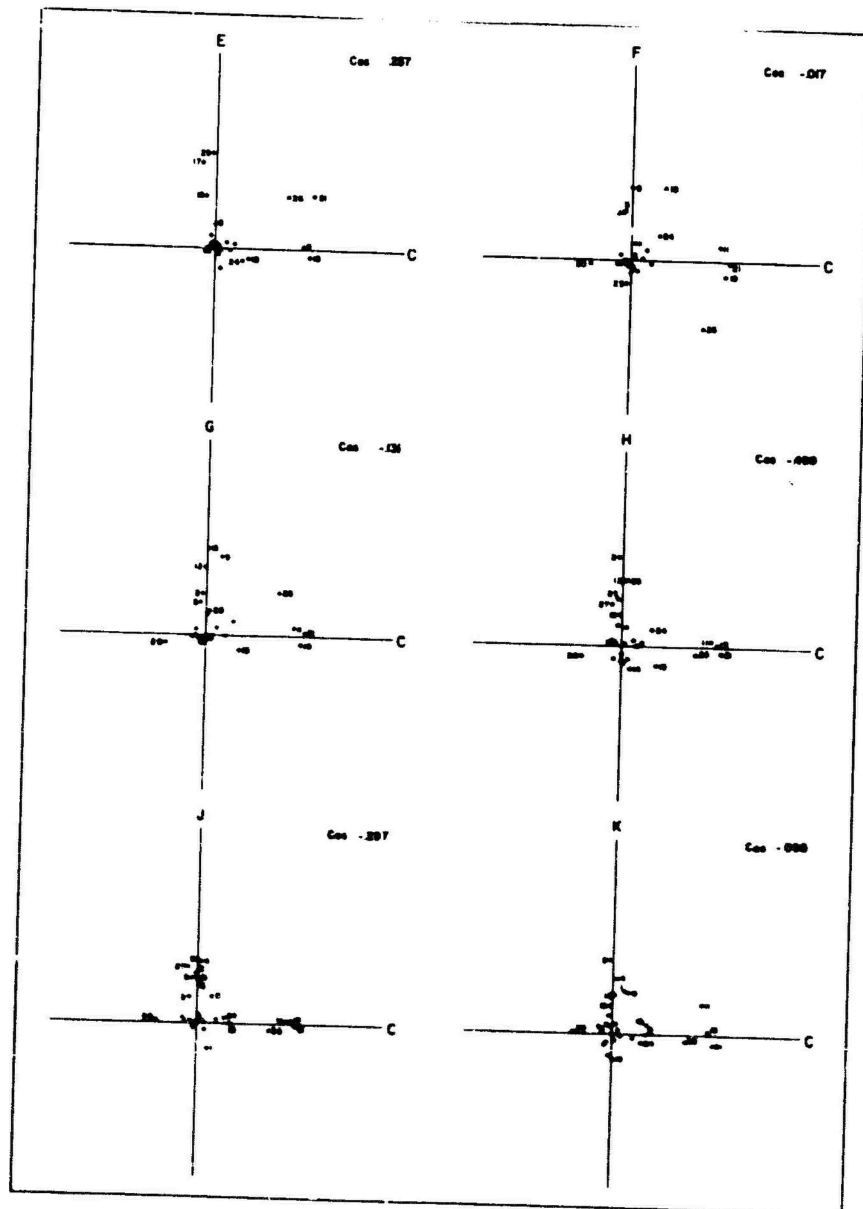


APPENDIX G

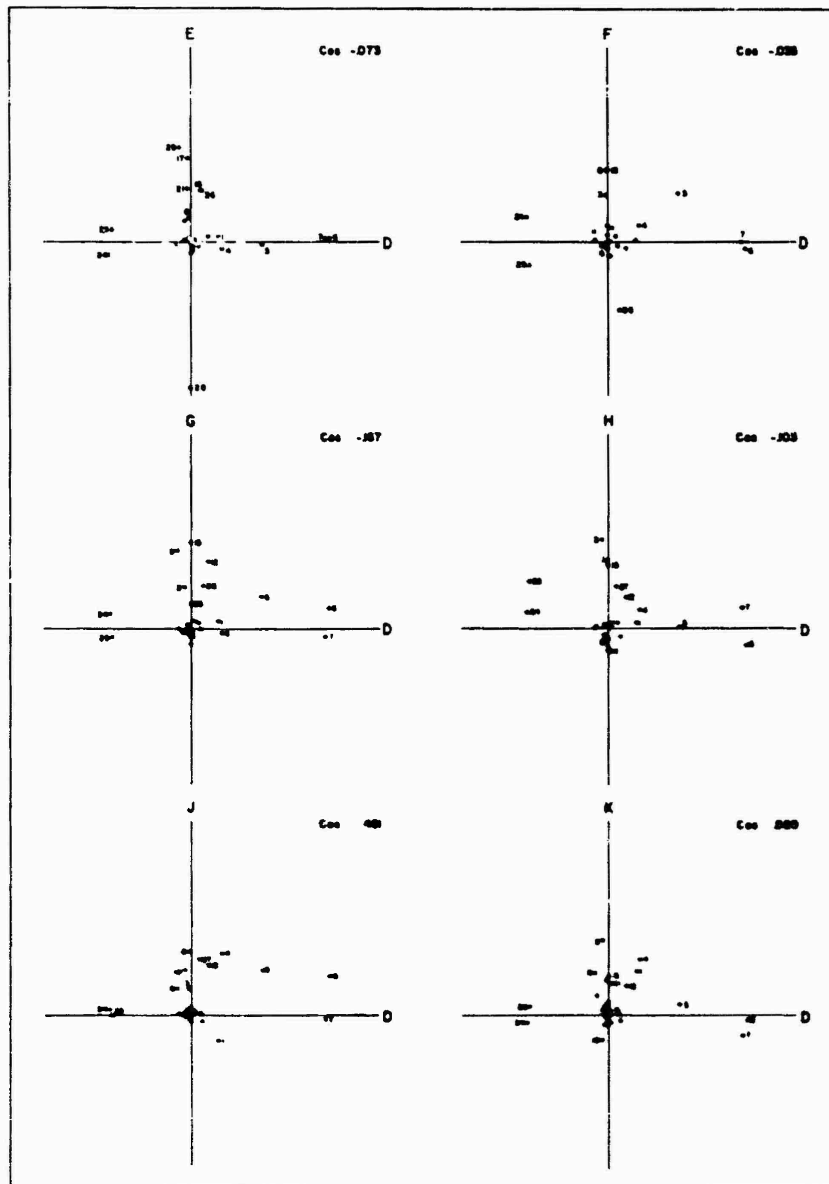
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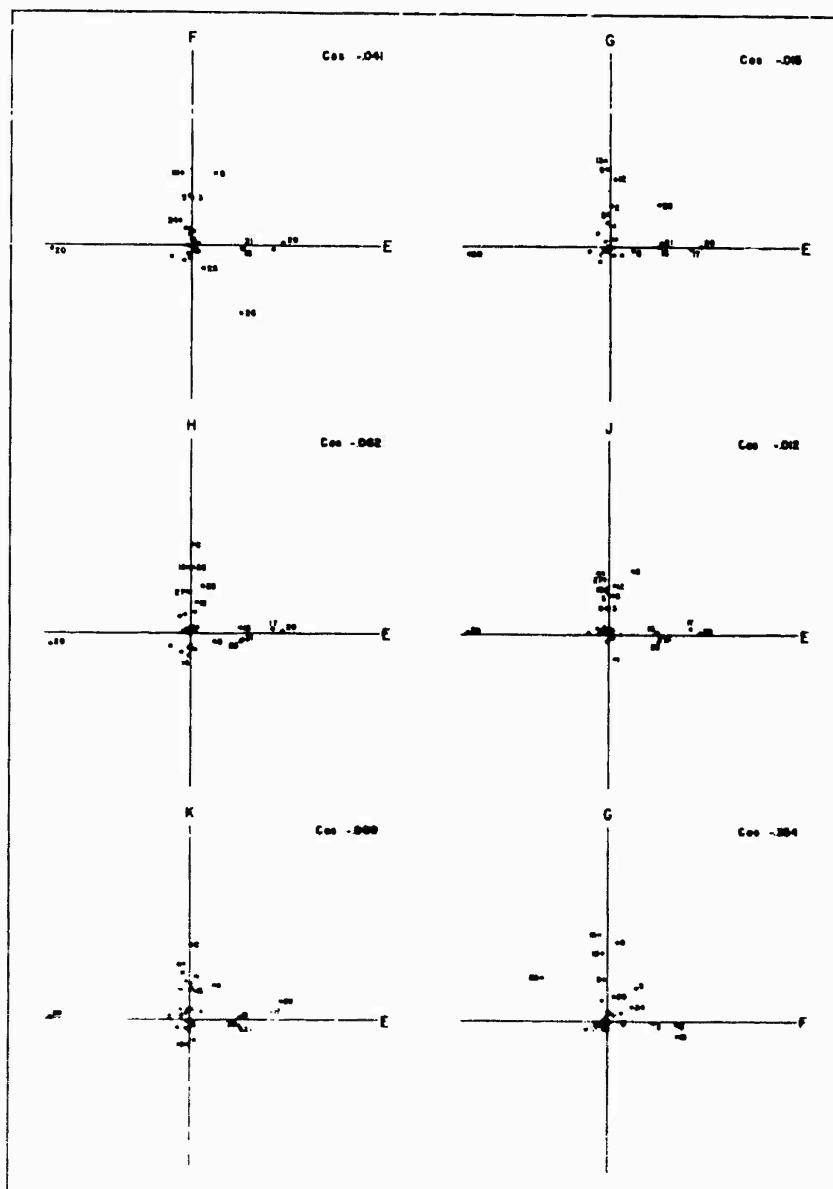
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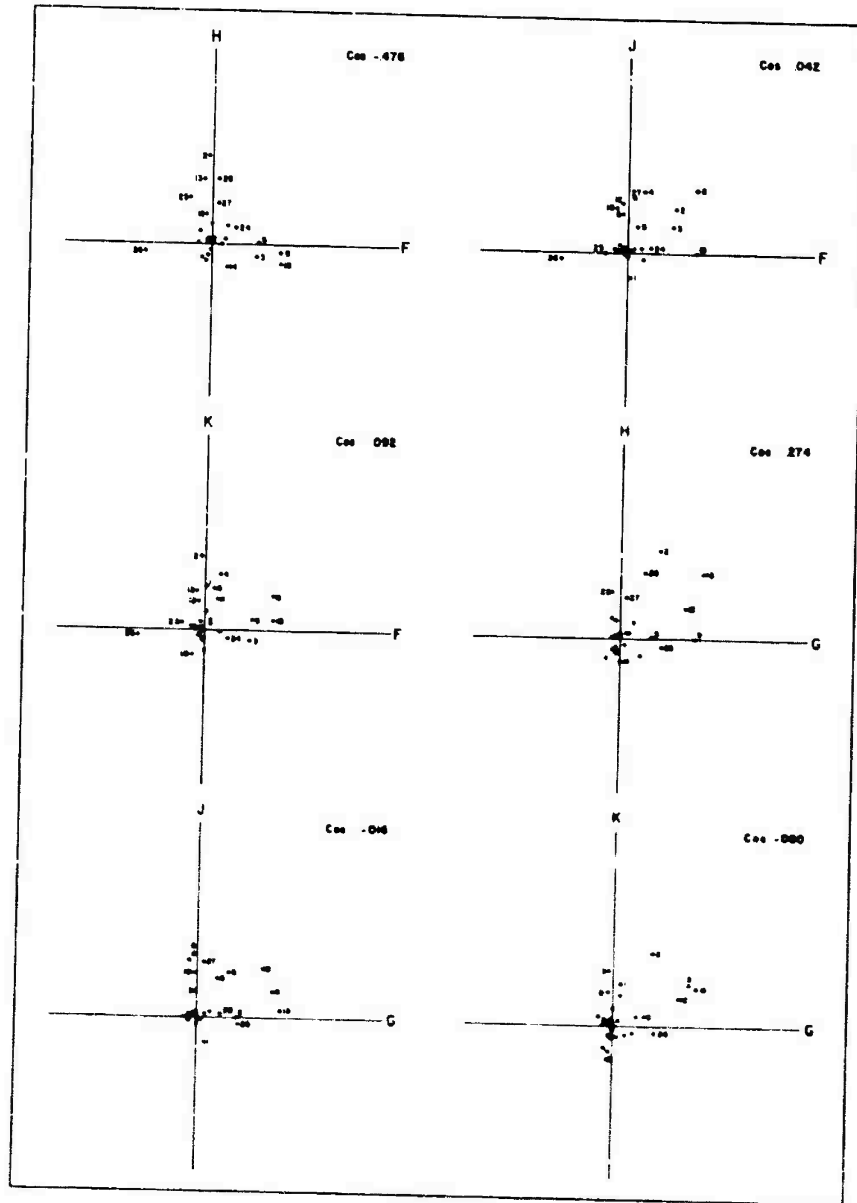
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A P P E N D I X G
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APPENDIX G
(continued)



APPENDIX G
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